DISCHARGE OF A GAS WITH SOLID PARTICLES FROM NOZZLES AT CRITICAL VELOCITIES

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Problems of the gas dynamics of two-phase flow with critical discharge conditions are considered. The flow parameters (pressure, density, and velocity of the gas) are calculated in one-dimensional approximation in convergent and cylindrical nozzles. The calculated data are compared with the experimental results obtained by investigating the discharge of a mixture of air and solid particles.

A method of calculating gas flows with solid particles in Laval nozzles in one-dimensional approximation is given in [1] and in two-dimensional approximation in [2]. When designing equipment for cladding sand-blasting devices, atomizers and nozzles for steam–water flows the problem frequently arises of calculating the parameters of a two-phase mixture discharging from convergent and cylindrical nozzles. The method of calculation in this case differs from the method of calculating the flow in a Laval nozzle. The special feature is that the velocity of the gas phase at the outlet from the nozzle cannot be greater than a finite (critical) value, corresponding to a Mach number of unity [4].

The problem of discharge of a two-phase mixture still has been little investigated. In [3], the discharge of an air–water mixture through a cylindrical nozzle was discussed. The authors of this paper assumed that a homogeneous mixture is moving in the nozzle with no velocity and temperature lag of the liquid particles. Calculations in [1, 2] show that even for very small particles (less than 3 μ) there is a considerable lag. Therefore, the experimental relations obtained in [3] require correcting when used for two-phase flow calculations with other parameters (for example, for flows with a coarser atomization of the liquid). A calculation and experimental data are given in [4] for the discharge of steam–water mixtures through a tube with a tube length to diameter ratio of more than 10. A two-layer model of the motion of a steam and liquid mixture was used for the calculation. It was assumed that the phase velocities vary along the tube length linearly and that there are no interaction forces between phases. These assumptions and the invalidity of the two-layer model at high degrees of dryness, when the particles of liquid are distributed uniformly in the cross section of the channel, may be one of the causes of discrepancy found by the author between the calculated and experimentally measured discharge rates.

A method is proposed in this present paper for calculating the parameters of a gas with solid particles, taking account of stagnation and interaction forces between the phases.

In order to calculate the flow, a model of a continuous medium was used, consisting of a gas phase and a "gas" of particles [5] having the following properties: the flow parameters depend on a single coordinate x; the gravitational force of the particles does not affect their motion; the volume occupied by the particles is negligibly small. In addition it was assumed that the particles are spherical and identical in size and that there is no heat exchange between particles and gas. The latter statement is valid for particles with a diameter of more than 10 μ [6]. (Particles of this and larger size are considered in the present paper.) The system of equations describing the flow of a two-phase continuous medium of a simulated gas with particles has the form

$$\frac{1}{v}\frac{dv}{dx}+\frac{1}{\rho}\frac{d\rho}{dx}+\frac{2}{r}\frac{dr}{dx}=0; \qquad (1)$$

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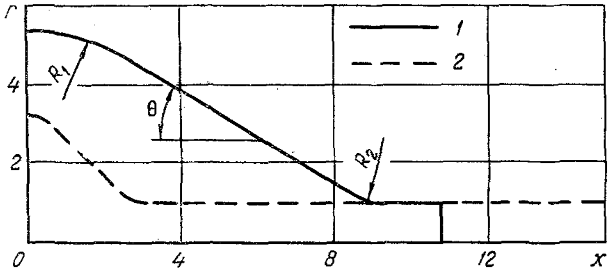

Fig. 1

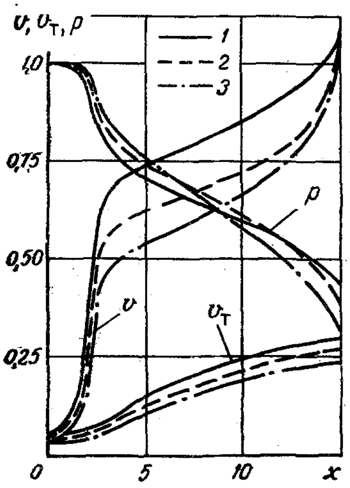

Fig. 2

Fig. 1. Profile of a convergent nozzle: 1) (R_1 = 5.0; R_2 = 2; θ = 30°) and the profile of a cylindrical nozzle; 2) (R_1 = 1; R_2 = 1; θ = 45°).

Fig. 2. Discharge of an air–alumina mixture (50 μ) through a cylindrical nozzle [1) μ = 1; 2) 2; 3) 4].

$$\frac{1}{v_т}\frac{dv_т}{dx}+\frac{1}{\rho_т}\frac{d\rho_т}{dx}+\frac{2}{r}\frac{dr}{dx}=0; \tag{2}$$

$$v\frac{dv}{dx}+\frac{1}{\rho}\frac{dp}{dx}+\frac{\rho_т}{\rho}f=0; \tag{3}$$

$$f=v_т\frac{dv_т}{dx}=\varphi(v-v_т); \tag{4}$$

$$v\frac{dh}{dx}-\frac{1}{\rho}v\frac{dp}{dx}+\frac{\rho_т}{\rho}f(v_т-v)=0; \tag{5}$$

$$\frac{1}{p}\frac{dp}{dx}=\frac{1}{\rho}\frac{d\rho}{dx}+\frac{1}{T}\frac{dT}{dx}; \tag{6}$$

$$h=\frac{k}{k-1}T;\ r=r(x);$$
$$\varphi=\frac{\pi}{8}c\frac{d^2}{m}\rho(v-v_т)\rho^*r^*. \tag{7}$$

Here (1) and (2) are the equations of continuity for the gas and particle phases; (3) and (4) are the equations of motion; (5) is the energy equation for the gas phase; (6) is the equation of state of the gas. All the quantities in the system (1)-(7) are represented in dimensionless form. Let r^*, v^*, and ρ^* be the characteristic dimensional constants for dimensions of length, velocity, and density. Then, in order to reduce to the dimensionless form the coordinate x is divided by r^*, the velocity by v^*, density by ρ^*, pressure by $\rho^*(v^*)^2$, enthalpy by $(v^*)^2$, temperature by $(v^*)^2/R$, and force by $(v^*)^2/r^*$ [2].

The drag coefficient of a spherical particle occurring in the expression for determining φ, takes the form proposed in [7]:

$$c=24\mathrm{Re}^{-1}+4\mathrm{Re}^{-\frac{1}{3}} \tag{8}$$

We take as characteristic dimensional constants: $r^*=r_e$, $\rho^*=\rho_0$, and $v^*=\sqrt{RT_0}$. The values of the flow parameters at the nozzle inlet have the form

$$p_0=T_0=\rho_0=1;\quad \rho_{т0}=\mathrm{const};$$
$$v_0=v_{т0}=M_0\sqrt{k}.$$

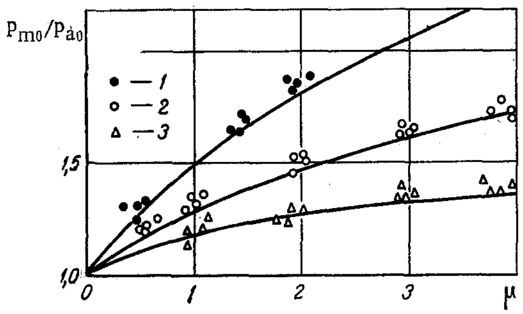

Fig. 3

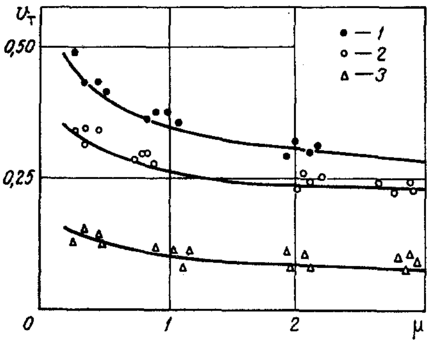

Fig. 4

Fig. 3. Discharge of a two-phase mixture through a cylindrical nozzle with $r_e = 2.5$ mm; air flow rate $1 \cdot 10^{-2}$ kg/sec; $T_0 = 295°C$ [1) air–graphite (10 μ); 2) air–alumina (50 μ); 3) air–corundum (150 μ)].

Fig. 4. Discharge through a convergent nozzle with $r_e = 1.5$ mm; air flow rate $5 \cdot 10^{-3}$ kg/sec; $T_0 = 295°K$ [1) air–graphite (10 μ); 2) air–alumina (50 μ); 3) air–corundum (150 μ)].

The calculation was carried out for $\rho_{T0} = 0.5$, 1, 2, and 4 for three types of mixture: air–graphite (average particle diameter 10 μ); air–alumina (50 μ) and air–corundum (150 μ). The value of M_0 was chosen such that in the outlet section M was equal to 1. This corresponded to "idling" conditions of the nozzle, for which a reduction of pressure in the surrounding medium did not lead to an increase of the flow velocity. In this paper, only this cycle has been considered. The function r(x) describing the nozzle profile is shown in Fig. 1.

The solution of systems (1)-(7) with the starting conditions of Eq. (9) was carried out by the Runge–Kutta method in a "Ural-2" electronic computer.

Figure 2 shows the change of basic parameters of a two-phase flow along the axis of a cylindrical nozzle.

In the experimental investigation, the flow of a two-phase mixture through nozzles with the profiles shown in Fig. 1, the relations between the pressure at the nozzle outlet, the velocity of the solid phase at the nozzle outlet and the concentration and size of the particles were studied. The particle velocity was determined from the expression for the total momentum of a two-phase jet [8]. The magnitude of the momentum was measured by means of a specially designed sensor, the operating principle of which is similar to that described in [9].

Figure 3 shows the calculated (continuous lines) and experimental data for the ratio of the pressure at the nozzle inlet p_{m0} with mixture discharge to the magnitude of the pressure p_{a0} in the case of discharge of pure air through the nozzle. The flow rates of the gas phase in both cases are identical.

The dependence of the velocity of the solid phase at the end of the nozzle on the concentration and size of the particles is shown in Fig. 4.

Comparison of the results of calculation and experiment permit the conclusion to be drawn concerning the acceptability of the proposed method of calculation of a two-phase flow. The small discrepancy between the calculated and measured values of pressure and velocity is explained, clearly, by the fact that no account is taken in the calculations of friction between the stream and the wall of the channel.

The analysis carried out of the flow of a gas with solid particles shows that an increase of the particle concentration and a decrease of particle size, whilst the flow rate of the gas phase is maintained, leads to a considerable increase of the initial pressure. The lag of the gas particles is stronger, the higher their concentration and the greater their diameter. Nozzle "idling" occurs at lower ratios of p_v/p_{m0} than for a pure gas.

NOTATION

x — is a coordinate directed along the axis of the nozzle;
v, v_T — are the velocity of gas and solid phase, respectively;

ρ, ρ_T are the density of gas phase and particle "gas";
p is the gas pressure;
f is the force acting per 1 kg of solid phase from the direction of the gas;
c is the drag coefficient;
m is the mass of particle;
d is the diameter of particle;
h is the enthalpy of gas;
T is the gas temperature;
R is the gas constant;
k is the isentropy index;
r is the radius of nozzle cross section;
Re is the Reynolds number;
M is the Mach number;
μ is the ratio of particle flow rate to gas phase flow rate.

Subscripts

0 denotes the flow parameter at nozzle inlet;
a denotes the pure air;
T denotes the solid phase;
e denotes the outlet section;
s denotes the ambient medium;
m denotes the mixture.

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